SEMI-ANNUAL STATUS REPORT ON NASA GRANT NGR-44-005-037

This is to submit a semi-annual status report covering the period from June 1, 1967 through November 30, 1967 on NASA Grant NGR-44-005-037. Individual reports are enclosed for each of the projects contained in the grant.

A PROBLEM CONCERNING A SPECIAL CLASS OF MONOTONE HOMOGENEOUS OPERATORS

This research effort has been concerned with problems involving diagonal products in nonnegative matrices. Results of Brualdi, Parter, and Schneider ("The diagonal equivalence of a nonnegative matrix to a stochastic matrix", J. Math. Anal. Appl. 16 (1966), 31-50) and of Sinkhorn and Knopp ("Concerning nonnegative matrices and doubly stochastic matrices", Pacific J. Math. 21 (1967), 343-348) show that corresponding to a given nonnegative square matrix A there is a unique doubly stochastic matrix of the form D_1AD_2 where D_1 and D_2 are diagonal matrices with positive main diagonals if and only if every positive element of A lies on at least one positive diagonal. Since the diagonal products of D_1AD_2 are proportional to the corresponding products in A, this research was partially concerned with finding just how well a doubly stochastic matrix is determined by its diagonal products.

The following results have been obtained by Sinkhorn and Knopp during the course of the research and have been presented in a paper "Problems involving diagonal products in nonnegative matrices" which will appear in the Transactions of the American Mathematical Society. If A is an $n \times n$ nonnegative fully indecomposable matrix whose positive diagonal products are equal, there exists a unique matrix B of rank one which is positive and is such that $b_{ij} = a_{ij}$ when $a_{ij} > 0$. It follows that no two doubly stochastic matrices have corresponding diagonal products proportional.

Though these interesting results have been found there remained several unanswered questions. Given two nonnegative $n \times n$ matrices A and B, under what conditions do there exist diagonal matrices D_1 and D_2 with positive main diagonals such that D_1AD_2 and D_2BD_1 are each stochastic? It is known that if for every zero submatrix $B[E|F]$ in B there is a zero submatrix $A[G|E]$ in A where F and G are disjoint, then there is a unique number $\mu > 0$ and diagonal matrices D_1 and D_2 with positive main diagonals such that D_1AD_2 and μD_2BD_1 are each stochastic.

The results obtained in the paper to appear in the Transactions have lead to consideration of the nearly decomposable matrix, i.e. a nonnegative square fully indecomposable matrix such that if any positive element is replaced by a zero, the resulting matrix is partly decomposable. It would be worthwhile to know the exact structure of such matrices. Necessarily if A is nearly decomposable there exist permutation matrices P and Q such that

$$PAQ = \begin{pmatrix} A_1 & 0 & 0 & \cdots & 0 & 0 & E_1 \\ E_2 & A_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & E_3 & A_3 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & E_{s-1} & A_{s-1} & 0 \\ 0 & 0 & 0 & \cdots & 0 & E_s & A_s \end{pmatrix}$$

where each E_k has exactly one positive element and each A_k is nearly decomposable. Unfortunately it is not sufficient for A to have this form. Just what is sufficient is still not answered.

Research will be continued toward the end of determining necessary and sufficient conditions on nonnegative matrices A and B so that D_1AD_2 and D_2BD_1 are simultaneously stochastic for appropriate D_1 and D_2 and of determining the structure of the nearly decomposable matrices. The results already obtained are most encouraging. They give a definite indication of a satisfactory completion of the problem.

RESEARCH IN ANALYSIS

DEFINITION

Suppose G is a lattice ordered group and $x \in G$. The statement that x is totally negative means that $x \leq e$ and if $h_1 \geq h_2 \geq h_3 \geq \ldots$ is a sequence of elements of G such that $x = \bigcup \{h_i\}_{i=1}^{\infty}$ then there exists an element $y \in G$ such that if n is a positive integer $y \geq \prod_{i=1}^{n} (h_i \cup e)$.

THEOREM

Suppose G is a lattice ordered group, S is a commutative sub semi-group of G with identity such that if K is a countable subset of S which is bounded above in G then K has a least upper bound in G which belongs to S, T is the commutative sub semi-group of G to which t belongs only in case t is the greatest lower bound in G of a countable subset of S, x is a totally negative element of T, and y is an element of T such that xy^{-1} belongs to T. Then there exists elements a and b of S such that $a \geq e$, $b \geq e$ and $ax \leq by \leq a$.

This result may lead to an improvement in the theory of order-preserving maps and integration processes. Research is continuing in this direction.

BASIC RESEARCH IN TOPOLOGY

Investigation into the relationship between the existence of various real-valued functions on a topological space and various topological properties of the space is continuing.

The following result has been achieved.

(1) If S is a pointwise paracompact, developable topological space then there is quasi-metrizable. That is, there is a real-valued function f on $S \times S$ such that $f(x,y) = 0$ iff $x = y$ and $f(x,y) + f(y,z) \geq f(x,z)$ for all x, y, and z in S.

Investigation is continuing on the problem as to whether each countably paracompact developable space is normal. It is conjectured that an example described by F. B. Jones [Summer Seminar, Wisconsin, 1965] offers such a space.

Partial results have been obtained which state equivalent problems to the one mentioned above.

The question of prime interest now is that of determining whether each countably paracompact developable space is hereditarily countably paracompact.

Further investigation will make heavy use of inverse limit systems. To be decided is whether an inverse limit system of proximity spaces produces a proximity space.

The problem of characterizing properties of compact continua by means of sequences of finite open covers of the continuum was considered. Indecomposability was thus characterized in the following theorem.

<u>Theorem</u>. A necessary and sufficient condition that the compact continuum M be indecomposable is that there exists a sequence $G_1, G_2, G_3, \cdots$ of finite open covers of M such that (1) for each positive integer n G_n is a coherent collection (2) for each n G_{n+1} is a strong refinement of G_n (3) for each n the mesh of G_n is less than $1/n$ (4) for each n G_n is an irreducible cover (5) M is the common part of $G_1^*, G_2^*, \cdots$ and (6) if i is an integer there is an integer $j > i$ such that if G_j is the sum of two coherent collections L_1 and L_2 then one of L_1^* and L_2^* intersects every open set in G_i.

ALGEBRAIC AND TOPOLOGICAL SEMIGROUPS

The research sponsored by this grant has dealt with the general problem of extending homological techniques to branches of mathematics other than those to which such techniques have previously been applied. In particular, we have considered the development of homological ideas in both the algebraic theory of semigroups and the theory of topological groups. In general, we are interested in the marriage of homological algebra and topological algebra.

One investigation along these lines has dealt with the notion of the tensor and torsion products of commutative semigroups. We proved, for example, that the tensor product $U \otimes V$ of two maximal subgroups U and V of commutative semigroups S and T, respectively, may be embedded in a natural way in the tensor product $S \otimes T$ of S and T. A definition of an exact sequence was given and it was proved that $\otimes$ is right-exact.

It was shown that the Grothendieck group of $S \otimes T$ is the tensor product of the Grothendieck group of S with the Grothendieck group of T. Several other theorems regarding the tensor and torsion products of semigroups were obtained. The results were submitted in the form of a paper to a mathematics research journal and preprints were made available to NASA.

In a slightly different direction, the investigator has been engaged in a joint research project in which homological techniques are being developed for the theory of topological groups. Several exciting results have already been proved. Since the paper we are writing is not yet complete we prefer to postpone the full report to a later date. May it suffice to say that some extremely nice results are available and that we plan to continue this investigation.

THE STRUCTURE OF NEAR-RINGS AND NEAR-RING AUTOMORPHISMS

The final report on this research was submitted May 31, 1967. Professor Joseph J. Malone, Jr. joined the faculty at Texas A & M University effective September, 1967.